

Gauge-invariant treatment of the integrated Sachs-Wolfe effect on general spherically symmetric spacetimes

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On the basis of the Gerlach-Sengupta theory of gauge-invariant perturbations, a formula of the integrated Sachs-Wolfe effect for a central observer is derived on general spherically symmetric spacetimes. It will be useful for comparative studies of theoretical and observational aspects of the integrated Sachs-Wolfe effect in the Lemaitre-Tolman-Bondi cosmological models which have been noticed by explaining the apparent acceleration without cosmological constant.

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I. INTRODUCTION

To explain the accelerating behavior of high-redshift supernovas, isotropic and homogeneous models with nonzero cosmological constant have been adopted as the standard ones[1–4], while we have another possibility to explain the cosmological acceleration as the apparent phenomenon in spherically symmetric inhomogeneous models[5–10]. Recently many theoretical studies for their inhomogeneous models have been done using the Lemaitre-Tolman-Bondi models[11–17].

While cosmological perturbations in isotropic and homogeneous models have fully been studied so far[18, 19], the treatment of perturbations in spherically symmetric inhomogeneous models has been formulated by Gerlach and Sengupta[20], and is being studied by several authors[21–23]. While the integrated Sachs-Wolfe (ISW) effect also in isotropic and homogeneous models has been often studied theoretically and observationally[24–31], it has been studied in inhomogeneous models only in a simple case such as being constructed with connected homogeneous regions[32].

In this paper we derive a formula of the integrated Sachs-Wolfe effect on general spherically symmetric inhomogeneous models on the basis of Gerlach and Sengupta's analysis for the perturbations[20]. In §2 we show the parity classification of perturbations in general spherically symmetric models and their notations. In §3 we consider the gauge-invariant perturbations of the null-geodesic equation and the null condition. In §4 we show a formula of the integrated Sachs-Wolfe effect in general inhomogeneous models. In §5 we consider the reduction to the isotropic and homogeneous models and show it can reproduce the well-known formula of the integrated Sachs-Wolfe effect in the Friedman models. §6 is devoted to the concluding remarks.

II. BACKGROUND MODEL AND LINEAR PERTURBATIONS

Following Gerlach and Sengupta's notation[20], the metric in general spherically symmetric spacetimes is expressed as

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\ &= g_{A,B} dx^A dx^B + r^2(x^C) d\Omega^2, \end{aligned} \quad (2.1)$$

where capital Latin indices A, B, C refer to x^0 and x^1 , small Latin indices a, b, c refer to x^2 and x^3 (or θ and ϕ), and $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$.

Linear metric perturbations in the above spacetimes are classified into odd-parity perturbations and even-parity perturbations. First, the odd-parity metric perturbations are expressed as @@

$$h_{\mu\nu} dx^\mu dx^\nu = h_A(x^C) S_a(\theta, \phi) (dx^A dx^a + dx^a dx^A) + h(x^C) (S_{a;b} + S_{b;a}) dx^a dx^b, \quad (2.2)$$

where the covariant derivative of the transverse vector harmonics S_a on the unit two-surface is indicated by colon $:$. The gauge-invariant quantities corresponding to h^A and h are

$$k_A = h_A - r^2(h/r^2)_{,A}. \quad (2.3)$$

Even-parity metric perturbations are

$$h_{\mu\nu} dx^\mu dx^\nu = h_{AB}(x^C) Y(\theta, \phi) dx^A dx^B + h_A(x^C) Y_{,a} (dx^A dx^a + dx^a dx^A) + r^2 [KY(\theta, \phi) \gamma_{ab} + GY_{,a;b}] dx^a dx^b, \quad (2.4)$$

and the corresponding gauge-invariant quantities are

$$\begin{aligned} k_{AB} &= h_{AB} - (p_{A|B} + p_{B|A}), \\ k &= K - 2v^A p_A. \end{aligned} \quad (2.5)$$

where $,A$ denotes the partial derivative, $|A$ denotes the two-dimensional covariant derivative with respect to x^A , and

$$v_A \equiv r_{,A}/r \quad \text{and} \quad p_A \equiv h_A - \frac{1}{2}r^2 G_{,A}. \quad (2.6)$$

III. NULL-GEODESIC EQUATIONS AND THE NULL CONDITION

Let us now assume that an observer is at the center of spherical symmetry and light emitted at the last scattering surface reaches the observer in the unperturbed state. Then the light path is radial and the wave vector $K^\mu (= dx^\mu/d\lambda)$ has the component $K^A (A = 0, 1)$ and $K^a (a = 2, 3) = 0$, where λ is the affine parameter. It satisfies the null-geodesic equation

$$K_{|B}^A K^B = 0 \quad (3.1)$$

and the null condition

$$K^A K_A = g_{AB} K^A K^B = 0. \quad (3.2)$$

Their perturbations satisfy the corresponding equations

$$\delta K_{,\nu}^\mu K^\nu + K_{,\nu}^\mu \delta K^\nu + \delta \Gamma_{\nu\lambda}^\mu K^\mu K^\nu = 0 \quad (3.3)$$

and

$$\delta g_{AB} K^A K^B + g_{AB} (\delta K^A K^B + K^A \delta K^B) = 0. \quad (3.4)$$

Similarly to the metric perturbations, the perturbation of wave vector, δK^μ is also classified into the odd-parity and even-parity perturbations.

The perturbed wave vector of odd-parity K_o is

$$\delta K_\mu dx^\mu = K_o S_a dx^a \quad (3.5)$$

and the wave vector of even-parity have two components K^e and K_A^e which satisfy

$$\delta K_\mu dx^\mu = K_A^e Y dx^A + K^e Y_{,a} dx^a. \quad (3.6)$$

For the infinitesimal gauge change of odd parity

$$\delta \xi_\mu^{\text{odd}} dx^\mu = M(x^C) [-(\sin \theta)^{-1} (\partial T / \partial \phi) d\theta + \sin \theta (\partial Y / \partial \theta) d\phi] \quad (3.7)$$

and that of even parity

$$\delta \xi_\mu^{\text{even}} dx^\mu = \xi_A(x^C) T(\theta, \phi) dx^A + \xi(x^C) [(\partial Y / \partial \theta) d\theta + (\partial Y / \partial \phi) d\phi], \quad (3.8)$$

we have

$$\begin{aligned} \bar{K}^o - K^o &= -r^2 (M/r^2)_{,A} K^A, \\ \bar{K}^e - K^e &= -r^2 (\xi/r^2)_{,A} K^A, \\ \bar{K}_e^A - K_e^A &= K_{,B}^A \xi^B - K^B \xi_{,B}^A. \end{aligned} \quad (3.9)$$

From these transformation property, we find the gauge-invariant quantity of odd parity

$$\Psi_o \equiv K^o + r^2 (h/r^2)_{,A} K^A \quad (3.10)$$

and gauge-invariant quantities of even parity

$$\begin{aligned}\Psi_e &\equiv K^e + K^B h_B, \\ \Phi_e^A &\equiv K_e^A - K_{|B}^A p^B + K^B p_{|B}^A,\end{aligned}\quad (3.11)$$

where p^A is given in Eq.(2.6).

Now let us derive the equations for the perturbed wave vectors by analyzing Eqs. (3.3) and (3.4). For odd-parity, we obtain from Eq. (3.3)

$$K_{,B}^o K^B + 2(r_{,A}/r)K^A K^o + (h_{A,B} - \Gamma_{AB}^C h_C)K^A K^B = 0. \quad (3.12)$$

By use of gauge-invariant quantities, this equation reduces to

$$r^2(\Psi_o/r^2)_{,A}K^A = -k_{A|B}K^A K^B, \quad (3.13)$$

where k_A is defined in Eq.(2.3). For K^o , we do not obtain any equation from the null condition Eq.(3.4).

For even parity, we obtain from the angular part ($\mu = a$) of Eq. (3.3)

$$K_{,A}^e Y^{|a} K^A + \delta\Gamma_{AB}^a K^A K^B + 2\Gamma_{bA}^a K^A K^{eb} = 0, \quad (3.14)$$

where

$$\begin{aligned}\Gamma_{bA}^a &= (r_{,A}/r)\delta_b^a, \\ \delta\Gamma_{AB}^a &= \frac{1}{2}Y^{|a}[-k_{AB} + h_{A|B} + h_{B|A}],\end{aligned}\quad (3.15)$$

so that we obtain

$$r^{-2}(\Psi_e r^2)_{,A}K^A = \frac{1}{2}k_{AB}K^A K^B. \quad (3.16)$$

The integration of this equation along the light path from the emitter (i) to the observer (f) leads to

$$(\Psi_e r^2)_f - (\Psi_e r^2)_i = \frac{1}{2} \int_{\lambda_i}^{\lambda_f} r^2 k_{AB} K^A K^B. \quad (3.17)$$

From the part $\mu = A$ of Eq.(3.3), we obtain similarly an equation for Φ_e^A

$$\Phi_{e|B}^A K^B + K_{|B}^A \Phi_e^B = -\frac{1}{2}g^{AB}(k_{DB|C} + k_{DC|B} - k_{BC|D})K^B K^C. \quad (3.18)$$

Moreover we obtain from Eq.(3.4)

$$\Phi_e^A K_A = -\frac{1}{2}k_{AB}K^A K^B, \quad (3.19)$$

where $K_A = g_{AB}K^B$. The second term of the left-hand side of Eq.(3.18) is rewritten using Eqs.(3.19) and (3.1-3.2) as

$$\begin{aligned}K_{|B}^A \Phi_e^B &= K_{|0}^A \Phi_e^0 + K_{|1}^A \Phi_e^1 \\ &= (K_{|0}^A - K_{|1}^A K_0/K_1)\Phi_e^0 - \frac{1}{2}k_{BC}K^B K^C K_{|1}^A/K_1 \\ &= -\frac{1}{2}k_{BC}K^B K^C K_{|1}^A/K_1,\end{aligned}\quad (3.20)$$

so that

$$\Phi_{e|B}^A K^B = -\frac{1}{2}[g^{AD}(k_{DB|C} + k_{DC|B} - k_{BC|D}) - k_{BC}K_{|1}^A/K_1]K^B K^C. \quad (3.21)$$

For $A = 0$, the left-hand side of this equation is rewritten again using Eqs.(3.19) and (3.1-3.2) as

$$(\Phi_{e,B}^0 + \Gamma_{BC}^0 \Phi_e^C)K^B = K^0(\Phi_e^0/K^0)_{,B}K^B + [K_{,B}^0/K^0 + \Gamma_{B0}^0 + \Gamma_{B1}^0 K^1/K^0]\Phi_e^0 K^B$$

$$\begin{aligned}
& -\frac{1}{2}k_{BC}K^BK^C\Gamma_{B1}^0K^B/K^1 \\
& = K^0(\Phi_e^0/K^0)_{,B}K^B - \frac{1}{2}K^BK^C\Gamma_{B1}^0K^B/K^1.
\end{aligned} \tag{3.22}$$

Therefore we obtain

$$(\Phi_e^0/K^0)_{,B}K^B = -\frac{1}{2K^0}[g^{0D}(k_{DB|C} + k_{DC|B} - k_{BC|D}) - k_{BC}(K_{|1}^0 + \Gamma_{D1}^0K^D)/K_1]K^BK^C. \tag{3.23}$$

Since $d(\Phi_e^A/K^0)/d\lambda = (\Phi_e^A/K^0)_{,B}K^B$, this equation leads to

$$(\Phi_e^0/K^0)_f - (\Phi_e^0/K^0)_i = -\frac{1}{2}\int_{\lambda_i}^{\lambda_f} d\lambda[g^{0D}(k_{DB|C} + k_{DC|B} - k_{BC|D}) - k_{BC}(K_{|1}^0 + \Gamma_{D1}^0K^D)/K_1]K^BK^C/K^0, \tag{3.24}$$

and by substituting this solution of Φ_e^0 to Eq.(3.19) we can obtain Φ_e^1 .

IV. INTEGRATED SACHS-WOLFE EFFECT

The temperature of the cosmic background radiation measured by an observer at the center can be written as

$$T_o = T_e/(1+z) = (\omega_o/\omega_e)T_e, \tag{4.1}$$

where z is the redshift of photons during their travel from the emitter e to the observer o and it is related to the emitted frequency ω_e and the observed frequency ω_o . The frequency ω measured by the observer with velocity U^μ is defined by

$$\omega = -g_{\mu\nu}U^\mu K^\nu, \tag{4.2}$$

where $K^\mu = dx^\mu/d\lambda$.

Now we consider the linear perturbations of frequencies, $\delta\omega$, from the background frequency $\bar{\omega}$. Then T_o/T_e is expressed as

$$T_o/T_e = \frac{\bar{\omega}_o + \delta\omega_o}{\bar{\omega}_e + \delta\omega_e} = \frac{\bar{\omega}_o}{\bar{\omega}_e} \left(1 + \frac{\delta\omega_o}{\bar{\omega}_o} - \frac{\delta\omega_e}{\bar{\omega}_e}\right). \tag{4.3}$$

Here we have

$$\begin{aligned}
\delta\omega &= -\delta g_{\mu\nu}\bar{U}^\mu\bar{K}^\nu - \bar{g}_{\mu\nu}\delta U^\mu\bar{K}^\nu - \bar{g}_{\mu\nu}\bar{U}^\mu\delta K^\nu \\
&= -\delta g_{AB}\bar{U}^A\bar{K}^B - \bar{g}_{AB}\delta U^A\bar{K}^B - \bar{g}_{AB}\bar{U}^A\delta K^B,
\end{aligned} \tag{4.4}$$

because the background quantities have only the components with $\mu = 0$ and 1. For δg_{AB} and δK^A , we have the gauge-invariant quantities k_{AB} and Φ_e^A . For the perturbation δU^A of the velocity field U^A , we can define the gauge-invariant quantity V^A by

$$V^A \equiv U_e^A - U_{|B}^A p^B + U^B p_{|B}^A \tag{4.5}$$

with

$$\delta U_\mu dx^\mu = U_A^e Y dx^A + U^e Y_a dx^a, \tag{4.6}$$

where U^A is the unperturbed velocity field, and U_A^e and U^e are the components with $\mu = A$ and a of δU^μ , respectively. Hence the gauge-independent counterparts of $\delta\omega$ consist of the following three counterparts

$$-k_{AB}\bar{U}^A\bar{K}^B, -\bar{g}_{AB}V^A\bar{K}^B, -\bar{g}_{AB}\bar{U}^A\Phi_e^B. \tag{4.7}$$

The change in frequencies caused by the matter perturbations during the photon travel from the last scattering surface to the present time is given by the last component which is called the ISW effect. It should be noticed that only Φ_e^A contributes to the ISW effect among the three components, while the other components (Ψ_o and Ψ_e) contribute to the gravitational lensing through angular deviation of light paths.

Since $\delta\omega/\omega = U_A \delta K^A / U_A K^A$, the gauge-invariant quantity representing the ISW effect, Θ , is expressed as

$$\Theta \equiv (U_A \Phi^A / U_A K^A)_f - (U_A \Phi^A / U_A K^A)_i. \quad (4.8)$$

In order to derive $U_A \Phi^A / U_A K^A$ unambiguously in arbitrary coordinate systems, we consider the inner products with a system of two orthonormal vectors $e_A^{(0)}$ and $e_A^{(1)}$, defined as $e_A^{(0)} \equiv U_A$ and $e_A^{(1)} \equiv n_A$, where n_A is the normal vector, i.e. $n_A U^A = 0$, $U_A U^A = -1$, $n_A n^A = 1$, $n_A = g_{AB} n^B$ and $U_A = g_{AB} U^B$. Moreover we define $e_{(B)}^A$ as $e_{(A)}^B e_B^{(C)} = \delta_A^C$. Then we can obtain from Eq.(3.21) in a similar manner to the derivation of Eqs.(3.22) and (3.23)

$$(\Phi_e^{(0)} / K^{(0)})_{,A} K^A = -\frac{1}{2K^{(0)}} [g^{(0D)} (k_{(DB|C)} + k_{(DC|B)} - k_{(BC|D)}) - k_{(BC)} (K_{(1)}^{(0)} + \Gamma_{(D1)}^{(0)} K^D) / K_{(1)}] K^{(B)} K^{(C)}, \quad (4.9)$$

where $K^{(A)} = e_B^{(A)} K^B$, $g^{(AB)} = e_C^{(A)} e_D^{(B)} g^{CD}$, $k_{(AB|C)} = e_{(A)}^D e_{(B)}^E e_{(C)}^F k_{DEF}$, and $\Gamma_{(BC)}^{(A)} = e_D^{(A)} e_{(B)}^E e_{(C)}^F \Gamma_{EF}^D$. Since we have $U_A \Phi^A / U_A K^A = \Phi^{(0)} / K^{(0)}$, $\Phi^{(0)} = e_A^{(0)} \Phi^A$ and $K^{(0)} = e_A^{(0)} K^A$, we obtain finally

$$\Theta = (\Phi^{(0)} / K^{(0)})_f - (\Phi^{(0)} / K^{(0)})_i = \int_{\lambda_i}^{\lambda_f} d\lambda [\Phi^{(0)} / K^{(0)}]_{,A} K^A. \quad (4.10)$$

Using Eqs. (4.9) and (4.10), therefore, we can derive the ISW effect quantitatively. Here we assume that there are no metric and matter perturbations at the emission epoch (i) and the observer epoch (f), to pick up only the integrated component of the Sachs-Wolfe effect caused by local inhomogeneities between these two epochs.

V. REDUCTION TO THE HOMOGENEOUS CASE

As a special case we consider here isotropic and homogeneous background models with the metric

$$ds^2 = a^2(\eta) [-d\eta^2 + d\chi^2 + \sigma^2(\chi) d\Omega^2] \quad (5.1)$$

with $\sigma(\chi) = \sin \chi, \chi, \sinh \chi$ for positive, null, negative curvature. In this case we have $x^0 = \eta$ and $x^1 = \chi$ and the orthonormal vectors are $e_{(0)}^A = U^A = (1, 0)/a$, $e_{(1)}^A = (0, 1)/a$, $e_A^{(0)} = (1, 0)a$ and $e_A^{(1)} = (0, 1)a$. The background wave vectors have the components $K^0 = -K^1 \propto a^{-2}(\eta)$, or $K^{(0)} = -K^{(1)} \propto a^{-1}$. In these models the gauge-invariant scalar-type perturbations are expressed as follows using the gauge-invariant Newton potential perturbation Φ_A and spatial curvature perturbation Φ_H (in Bardeen's notation)[18, 19].

$$\begin{aligned} k_{(00)} &= k_{00}/a^2 = -2 \int d\mathbf{k} \Phi_A Q(\chi), \\ k_{(01)} &= k_{01}/a^2 = 0, \\ k_{(11)} &= k_{11}/a^2 = 2 \int d\mathbf{k} \Phi_H Q(\chi), \\ k &= \int d\mathbf{k} \Phi_H Q(\chi), \end{aligned} \quad (5.2)$$

where \mathbf{k} is the wave number and the scalar harmonics $Q^{(0)}$ are expressed as

$$Q^{(0)} = Q(\chi) Y(\theta, \phi) \quad (5.3)$$

in terms of spherical harmonics $Y(\theta, \phi)$. Then we obtain from Eqs (4.9) and (4.10)

$$\begin{aligned} \Theta &= -\frac{1}{2} \int d\mathbf{k} \int_{\lambda_i}^{\lambda_f} I K^{(0)} d\lambda, \\ aI &= 2(\Phi_A + \Phi_H)_{,\eta} Q - 4\Phi_A Q_{,\chi}. \end{aligned} \quad (5.4)$$

Here $K^0 d\lambda = d\eta$ or $K^{(0)} d\lambda = a d\eta$. Since $\chi + \eta = \text{const}$ along radial light paths, we obtain $\int \Phi_A Q_{,\chi} d\eta = \int [-\frac{d}{d\eta}(\Phi_A Q) + \Phi_{A,\eta} Q] d\eta$, so that

$$\int_{\eta_i}^{\eta_f} aI d\eta = \int_{\eta_i}^{\eta_f} [2(\Phi_H - \Phi_A)_{,\eta} Q + 4\frac{d}{d\eta}(\Phi_A Q)] d\eta. \quad (5.5)$$

From Eq. (5.4) we obtain

$$\Theta = \int d\mathbf{k} \int_{\eta_i}^{\eta_f} (\Phi_A - \Phi_H)_{,\eta} Q d\eta. \quad (5.6)$$

It should be noticed here that another term $2 \int d\mathbf{k} (\Phi_A Q)|_i^f$ in Eqs. (5.4) and (5.5) vanishes, because the influence from metric perturbations at the emission and observer epochs are neglected. The above result is well-known as the ISW effect in the Friedman models[33].

VI. CONCLUDING REMARKS

We derived a formula of the ISW effect for a central observer which is appropriate to studying the ISW effect caused by local inhomogeneities like galaxies, clusters and voids between the emission and observer times. The next step is to extend this formula to the case of off-central observers, which will be more complicated because of the additional asymmetry.

When the metric perturbations in realistic Lemaitre-Tolman-Bondi models will be explicitly obtained, our formula will be useful to derive the ISW effect in them theoretically and compare it to the observational one. As another next step, we shall attempt to derive it in a case of self-similar spacetimes.

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